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Project Goal: Curvatures $< T$

- given that T = some integer and packing P exists, what is the number of circles in packing P less than T

∴ we went into averages,

averages morph into T!

-1, 2, 2, 3

So... time to calculate the averages →

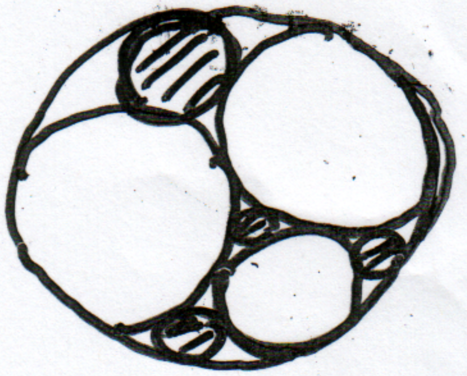
step a) how many circles are there in each generation?

0th generation
(4 circles)

1st 3 6 6 15

6 6 15 11 14 23 11 14 23 38 35 38

...etc.



...

gen	newborn	ttl
0	4	4
1	4	8
2	12	20

$\Rightarrow$ $4 \times 3^{n-1}$ newborn/gen or $2 + 2 \times 3^{n}$ ttl

$-1, 2, 2, 3$

Step b) the hard way...

0th gen $\rightarrow \dfrac{-1+2+2+3}{4} = 3/2$

1st gen $\rightarrow \dfrac{15+6+3+6}{4} = 15/2$

2nd gen $\rightarrow \dfrac{26+26+47+47+35+35+30+30+39+39+18\ldots+102}{12} = 99/2$

3rd gen $\rightarrow$ ...no...

Step b) the 'easy way'... programming!

allcurvatures(a, b, c, d, n)

$\rightarrow$ vector of all curvatures of circle packing that has 0^{th} gen [a, b, c, d] up to the n^{th} generation.

avgcurgen(a, b, c, d, n)

$$\rightarrow \frac{\text{sum(allcurvatures}(a,b,c,d,n)) - \text{sum(allcurvatures}(a,b,c,d,n-1))}{4 \cdot 3^{n-1}}$$

= average curvature of generation n.

We've got the averages — what do we plot them against?

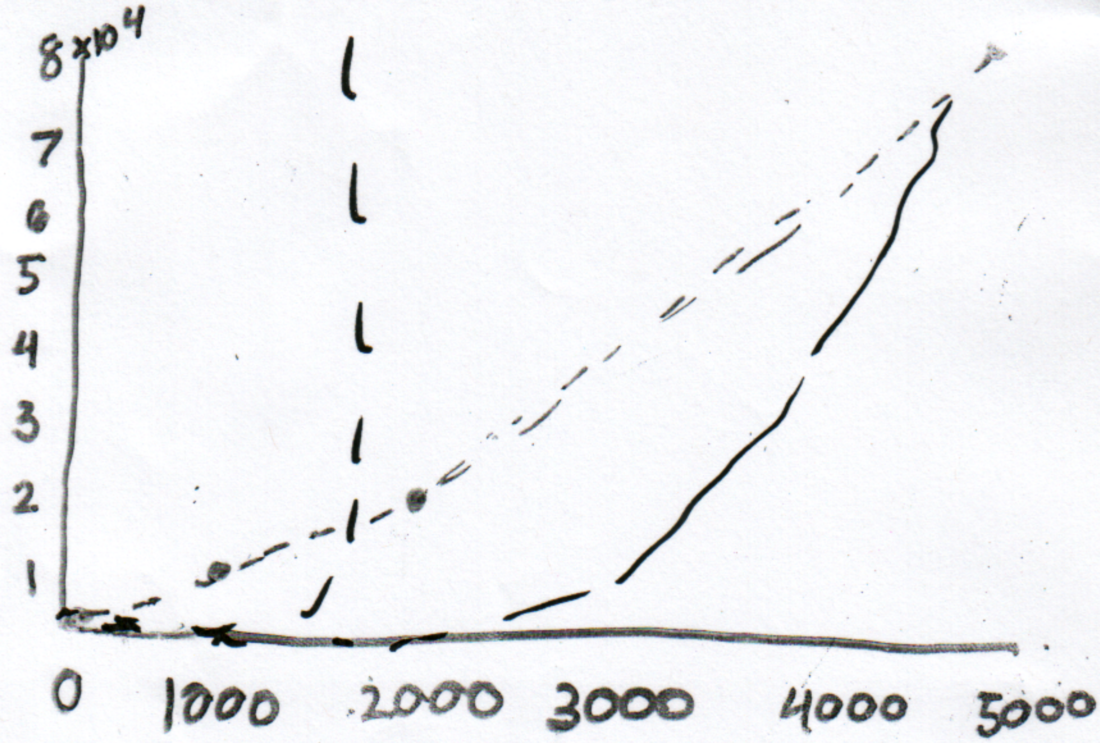
- Since we're trying to find the number of curvatures < T, we need to find the "number of curvatures under each average."

For the average of generation n, that is the number of curvatures in the generations below n, and <u>half</u> the curvatures "born" in generation n.

$\Rightarrow$ the y-coordinate for $x = a_n$ is

$$2 + 2\cdot 3^{n-1} + \frac{4\cdot 3^{n-1}}{2} = \underline{\underline{2 + 4\cdot 3^{n-1}}}$$

So we plotted a_n vs. $2 + 4\cdot 3^{n-1}$ for $n = 1$ to 8 and got....



ignoring my poor drawing skills / graphing skills... it's exponential!